AMC 8 Problem Solving Seminar

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Problem 1

Consider the set of all fractions $\frac{x}{y}$, where x and y are relatively prime positive integers. How many of these fractions have the property that if both numerator and denominator are increased by 1, the value of the fraction is increased by 10%?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinitely many

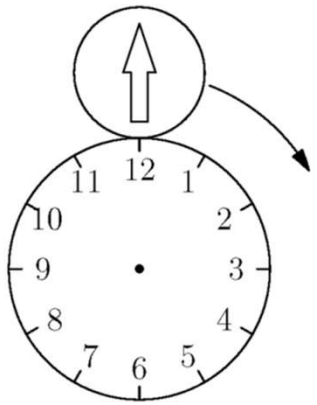
Problem 2

If $y + 4 = (x - 2)^2$, $x + 4 = (y - 2)^2$, and $x \neq y$, what is the value of $x^2 + y^2$?

(A) 10 **(B)** 15 **(C)** 20 **(D)** 25 **(E)** 30

Problem 3

The diagram below shows the circular face of a clock with radius 20 cm and a circular disk with radius 10 cm externally tangent to the clock face at 12 o'clock. The disk has an arrow painted on it, initially pointing in the upward vertical direction. Let the disk roll clockwise around the clock face. At what point on the clock face will the disk be tangent when the arrow is next pointing in the upward vertical direction?



(A) 2 o'clock **(B)** 3 o'clock **(C)** 4 o'clock **(D)** 6 o'clock **(E)** 8 o'clock

Problem 4

Hexadecimal (base-16) numbers are written using the numeric digits 0 through 9 as well as the letters A through F to represent 10 through 15. Among the first 1000 positive integers, there are n whose hexadecimal representation contains only numeric digits. What is the sum of the digits of n?

(A) 17 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** 21

Problem 5

The isosceles right triangle ABC has right angle at C and area 12.5. The rays trisecting $\angle ACB$ intersect AB at D and E. What is the area of $\triangle CDE$?

(A) $\frac{5\sqrt{2}}{3}$ **(B)** $\frac{50\sqrt{3}-75}{4}$ **(C)** $\frac{15\sqrt{3}}{8}$ **(D)** $\frac{50-25\sqrt{3}}{2}$ **(E)** $\frac{25}{6}$

Problem 6

A rectangle has area A cm^2 and perimeter P cm, where A and P are positive integers. Which of the following numbers cannot equal $A + P$?

(A) 100 **(B)** 102 **(C)** 104 **(D)** 106 **(E)** 108

Problem 7

Among the positive integers less than 100, each of whose digits is a prime number, one is selected at random. What is the probability that the selected number is prime?

(A) $\frac{8}{99}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{9}{20}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{9}{16}$

Problem 8

Let a, b, and c be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation $(x-a)(x-b)+(x-b)(x-c)=0$?

(A) 15 **(B)** 15.5 **(C)** 16 **(D)** 16.5 **(E)** 17

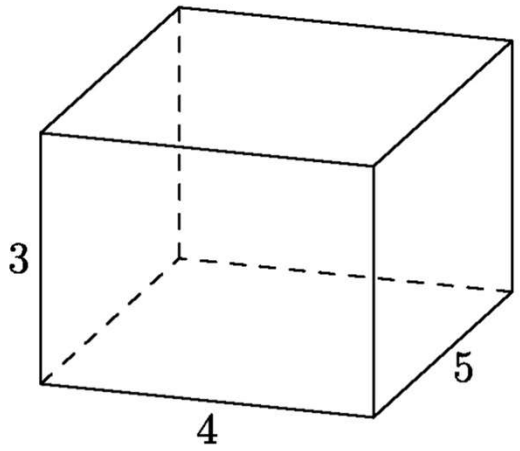
Problem 9

The town of Hamlet has 3 people for each horse, 4 sheep for each cow, and 3 ducks for each person. Which of the following could not possibly be the total number of people, horses, sheep, cows, and ducks in Hamlet?

(A) 41 **(B)** 47 **(C)** 59 **(D)** 61 **(E)** 66

Problem 10

The centers of the faces of the right rectangular prism shown below are joined to create an octahedron. What is the volume of the octahedron?



(A) $\frac{75}{12}$ **(B)** 10 **(C)** 12 **(D)** $10\sqrt{2}$ **(E)** 15